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Sonological and psychoacoustic Characteristics of Carillon Bells

Abstract

Sounds recorded from carillon bells are subjected to sonological analysis in regard of spectral and temporal parameters such as distribution and ratios of spectral components, spectral centroid, decay of components, and modulation phenomena. Detailed analyses of bell sounds reveal some common features as well as characteristics peculiar to certain bells and bell founders.
Since bell sounds are in part inharmonic, and often have a rich spectrum, perception of pitch and timbre of such complex sounds can be difficult even for musically trained listeners. Some findings from experiments making use of bell sounds as well as music played on carillons are reported, and are discussed with respect to psychoacoustics.

1. Introduction

Vibrational behaviour and sound characteristics of church and carillon bells as well as their tuning have been investigated since long (see Lehr 2000a). In the 20th century, development of electroacoustic measurement devices allowed for more detailed analyses of modes of vibration found in bells as well as of components contained in the sound recorded from swinging and carillon bells (e.g., van Heuven 1949, Slaymaker & Meeker 1954). Also, from such measurements, hypotheses as to the original tuning of historic carillons have been made (e.g., Lehr 1951).
During the past decades, in psychoacoustics phenomena subsumed under the heading of „virtual pitch" have been investigated in depth (see de Boer 1976, Terhardt 1998). By virtual pitch, a pitch sensation caused by the interplay of several (harmonic and/or inharmonic) spectral components is understood. The most basic virtual pitch is that of the „missing fundamental". In sounds composed of a number of consecutive harmonics (e.g, harmonics no. 4, 5, 6) yet lacking the fundamental, the pitch perceived will still correspond to that of a fundamental frequency, f_1. The reason simply is that the frequency with which the complex waveshape resulting from superposing harmonics 4, 5 and 6 repeats per second equals the frequency of the fundamental. Fitting a „pseudo-fundamental" to a complex waveshape comprising only higher partials (the so-called „residue", see Schouten 1940) is still possible in case the component frequencies deviate somewhat from harmonic ratios (see Schneider 1997, 145ff., 2000). If, however, the frequencies of the components fall off too much from harmonic ratios, this matching fails since the complex waveshape lacks clear periodicity. In general, an increase in inharmonicity of spectral components of a sound means that the periodicity of its time signal degenerates, and becomes more and more difficult to detect. This is a problem bearing also to pitch perception in bell sounds (see below).
The concept of „virtual pitch" that had been advanced by J.F. Schouten, in 1940, not only underpinned the correlation of the „residue" and periodicity yet also explained the strike note (Dutch *slagtoon*, German *Schlagton*) of bells as an

auditory phenomenon resulting from the interplay of spectral components. Drawing to the typical minor-third bell spectrum as found in bells cast by F. and P. Hemony, Schouten argued that the strike note can be attributed to the combination of the components no. 5, 7, and 10 that are conceived as being parts of a harmonic series (Schouten 1940, 293):

1	2	3	4	5	6	7	8	9	10
C	c	e^b	g	c'	e'	g'	a'	h'	c''

The strike note in this case should be c, and thus equivalent to c, the prime of the bell's characteristic tones. Schouten's approach to the strike note as a virtual pitch has become influential for later research (e.g., Schouten & t'Hart 1965, Bruhn 1980, Terhardt & Seewann 1984, Schad & Frik 1994) which confirmed Schouten's view.

2. Fundamental vibroacoustics of bells

From an acoustical point of view, many bells basically can be considered as shells that approximate a cylinder closed at one end and open at the other. With such a cylindrical shape, a bell ideally should be axisymmetrical, and mass distribution thus would be even around the middle axis x of the cylinder of radius r. However, the geometry of most of the historical swinging bells found in many countries in Europe as well as bells that are used in carillons is more complex in that the radius r, and hence the diameter d of the bell vary along its middle axis $A - A'$ (see figure 1) as does the thickness h of the bell's wall. Thereby, also the bending stiffness B varies with respect to the bell profile. At the so-called soundbow (or ring), stiffness is large due to the thickness of the bell's wall at this point whereas at the so-called shoulder of the bell, stiffness again is considerable yet has to be attributed to the shape itself, namely the circular plate adjacent to the shoulder which closes the bell, and carries the elements necessary to fasten a hanging bell to a support (see figure 1). Further, thickness of the wall and stiffness are factors that bear to the internal damping (attributed to stress in the material) of the bell. Because of these features, the vibration theory for this type of compound shell that has also be conceived of as several ring-like segments added on top of each other along $A - A'$, is very complex (as is the vibration theory for shells in general; see Rayleigh 1877/1945, Flügge 1962, Kalnins & Dym 1976, Göldner 1985, Leissa 1993b).
Historically, the development of the theory of vibration for bells did stem from calculations Leonard Euler provided for vibrating rings. He viewed a bell as a series of „annuli elementares" (see Euler 1776/1957). Lord Rayleigh (1877/1945, Vol. 1, §§ 232-235), on the basis of a cylindrical shell taken as a curved plate, gave a detailed account of the modes of vibration found in a typical bell with (almost perfect) rotational symmetry. He considered flexural vibrations around the bell's circumference (the zero points of which result in nodal meridians at equal distances), and along the bell's axis (the zeros of which result in nodal circles). Nodal meridians and nodal circles divide the surface into segments. The number of nodal meridians and nodal circles defines the number of segments which vibrate opposite in phase to each other (see cf. Grützmacher, Kallenbach & Nellessen 1965/66).
Since the corpus of a typical western bell consists of a massive ring near the

mouth plus a more or less cylindrical shell (i.e., the waist) added to it, certain modes of vibration appear to be „ring driven“ while others are regarded as being „shell driven“. Modes of (flexural) vibration have been analyzed with various methods, and have been described and classified in great detail (cf. Grützmacher et al. 1965/66, Perrin & Charnley 1973, Perrin, Charnley & DePont 1983, Perrin et al. 1985, Rossing & Sathoff 1980, Rossing & Perrin 1987, Lehr 1986, Fletcher & Rossing 1991/1998, ch. 21.1, Fleischer 1996).
As with other vibrating systems, for the flexural motion one has to distinguish between inextensional and extensional modes of vibration (see Rayleigh 1877/1945, § 232), the latter involving stretching of the bell corpus whereas for inextensional modes a neutral ring for each radial plane can be assumed.
Regarding vibrations of shells such as cylinders where axial, radial and tangential displacement of particles occurs, usually some simplifications have to be made to establish equations of motion, and to calculate eigenvalues and eigenfrequencies, respectively (see, e.g., Grützmacher et al. 1966, Leissa 1993b, Heckl 1997, 743). It should be mentioned that, in the terminology used by Charnley and Perrin (1975) in regard of bells, axial motion is labelled „meridian-tangential“, and tangential is specified as „ring-tangential“.
It has been shown experimentally that in bells also modes of pure torsional vibration about the symmetry axis can occur (Charnley & Perrin 1975). Torsional („twisting“) vibration is well known from rods. In such structures as well as in plates, another type of vibration is found usually labelled „quasi-longitudinal“ (see Cremer & Heckl 1967, 78ff.). Quasi-longitudinal waves involve strain/stress of the material along the x-axis as well as contraction of the cross section whereby particles in a bar are displaced also in the direction of the y- and the z-axis, respectively (in a plate, this motion is almost restricted to the z- dimension); therefore, phase velocity c_D for such waves is considerably smaller than that of pure longitudinal waves. It seems reasonable that quasi-longitudinal (strain) waves might occur also in bells along their wall, in particular in the waist. There are some indications for such types of vibration in historical bells; in one instance, a mode of vibration was reported having a frequency close to that of the nominal, and being strong enough to interfere with that partial (see Lehr 1986, p. 2004).
Investigations of bells are mostly confined to types of flexural vibration because these cause motion normal to the bell's surface. Since the walls of the bell couple directly to the sound field, motion normal to the surface will radiate most of the sound that becomes audible. Since efficient radiation of bending waves from the vibrating surface of a bell requires that the wave speed C_B must be at least equal to, or greater, than the sound speed in air (340 m/s; see Cremer & Heckl 1967, 457ff.), it follows that modes of vibration of higher order can be more prominent in the spectrum than those of lower order. (As to aspects of sound radiation from bells including damping, see Van Heuven 1949, Grützmacher et al. 1965/66, Rossing & Sathoff 1980, Fletcher & Rossing 1991/1998, ch. 21.11). The most important partials of the bell's sound spectrum stem from flexural vibrations.
Among the methods that have been employed to investigate normal modes of vibration, and to visualize patterns of vibration in bells, are time-averaged hologram interferograms (Rossing & Sathoff 1980), Finite Elements (*FEM*, see Perrin, Charnley & DePont 1983; Schad & Frik 1993a), and modal analysis (MA, see Fleischer 1996). The normal modes of the bell are described analogous to normal modes of a circular plate that vibrates in segments defined by nodal

meridians (m), and nodal circles (n).

The modes of vibration typical for „octave minor third bells", that is, for bells which have the interval of a perfect (or nearly so) octave between hum and prime as well as between prime and nominal, and a strong minor third above the prime (see Lehr 1986, 1991) are listed in table 1:

Table 1: scheme of partials and respective modes of vibration

name	frequency ratio to prime	mode (nodal meridians *m*; nodal circles *n*)
hum	0.5	4, 0 [2, 0; see discussion below]
prime [fundamental]	1	4, 1 [2, 1]
tierce [minor third]	1.2	6, 1 [3, 1]
quint [fifth]	1.5	6, 1 [3, 1]
nominal [octave]	2	8, 1 [4, 1]
tenth [major third]	2.52	8, 1 [4, 1]
twelfth [fifth]	3	10, 1 [5, 1]
thirteenth [major sixth]	3.36	10, 1 [5, 1]
double octave	4	12, 1 [6, 1]
upper fourth	5.33	14, 1 [7, 1]
upper major sixth	6.73	16, 1 [8, 1]
triple octave	8	18, 1 [9, 1]
minor third	9.5	20, 1 [10, 1]
fourth	10.68	22, 1 [11, 1]
fifth	12	24, 1 [12, 1]
major sixth	13.45	26, 1 [13, 1]

In most cases, these partials that have from 4 to 26 nodal meridians but only one nodal circle (or none, as is the case with the the hum), will form the strongest peaks in spectra obtained from actual bell sounds recorded in the free field. The phenomenon that there are pairs of modes that have identical numbers of meridians and circles yet represent different partials (for example, 6, 1 [3, 1], comprising the minor third and the fifth above the fundamental), stems from the fact that the position of the nodal circle on the bell's surface can vary considerably. The nodal circle for the minor third (6, 1) will be found on the waist while the nodal circle for the fifth (6, 1) is located near the soundbow of the bell (cf. Fletcher & Rossing 1991/1998, ch. 21.1).
Even though the vibrational modes listed in table 1 in many cases will be the most prominent ones, other modes can be found like, for example, the eleventh (ratio to the prime/fundamental: 2.67) that has six half-meridians that are equivalent to three full meridians conceived of as extending over the crown of the bell, and two nodal circles (Schad & Frik 1993). In measurements based on acoustical excitation of an English church bell, no less than 134 partials representing modes of vibration have been found in the frequency range from 292 Hz (hum note) up to 9300 Hz (Perrin et al. 1983). Of course, it is not possible to relate all these modes to partials that constitute sections of harmonic series such as will be found in table 1. In fact, actual bell sounds often contain many more inharmonic components than just the minor third that in many bells is strong in amplitude, and thereby is

characteristic of most of our church and carillon bells (see table 2 below). The minor third spectral component can interfere with the major third (tenth) partial that is found one octave higher. The spectral composition and in part inharmonic sound of minor-third bells at times have been found unsatisfactory to perform musical pieces written in a major key on a typical minor-third carillon. This issue played a role in the development of major third bells that, to be sure, have a much different geometry and shape of the wall's profile (see Lehr et al. 1987).

Due to the bell's axial (rotational) symmetry, modes of flexural vibrations with m > 2 are found in doublets (in German terminology called *Zwillingstöne*). This means that such modes occur in nearly degenerate pairs (cf. Perrin & Charnley 1973). In vibrating systems, two (or more) eigenfunctions and eigenmodes that produce the same eigenfrequency are called degenerate. This condition is often met in quadratic membranes and plates where ω_{mn} can be the same as ω_{nm} (e.g., $\omega_{21} = \omega_{12}$; see Gehrtsen/Vogel 1993, 176).

In case the rotational symmetry of the bell around its middle axis *A* – *A*' would be perfect, the meridians of one member of the degenerate pair would be found lying exactly on the vibration antinode of the other. Since especially historical swinging bells rarely have been cast to result in perfect mass symmetry, and may exhibit both variations in the thickness along the wall as well as deviations from a perfect ring with respect to the cross section, the two members of the pair have different vibration frequencies. In the spectrum of the bell sound one therefore quite often finds twin peaks representing the two members of a (nearly) degenerate pair. The distance of the peaks increases with the amount of deviation from perfect axial symmetry. Typically, the frequency difference of the two members of such a pair is a few Hertz (in cents, the difference often is less than 30 cents), and perceptually the amplitude modulation caused thereby will be registered as beats or roughness. In case the effect is more drastic, it usually is labelled *warble* (cf. Perrin & Charnley 1973, Lehr 2000b). There are indications that bell founders of the past deliberately may have created deviations from symmetry; eccentric shapes where degenerate pairs are separated into two independent components, possibly were employed to reduce warble.

As with all idiophones, flexural vibrations in bells result in bending waves, and hence in principle will produce inharmonic spectra because of frequency dispersion (that is, $C_B \sim \sqrt{f}$; $\lambda \sim 1/\sqrt{f}$). However, the profile of the bell as it has been developed during the Middle Ages to result in the prototypical *minor third bell* yields spectral components many of which correspond to partials of a harmonic spectrum. In general, a number of the spectral components can be assigned to several harmonic series that are, however, incomplete. Also, there are components that have inharmonic frequency relations both with the prime (fundamental), and among each other. Further, because of the profile necessary to produce the minor third, the octaves even in bells designed as so-called octave bells are hardly perfect, and rather tend to yield some characteristic deviations (cf. Lehr 1991). The same holds true with respect to historical carillon bells where frequencies of spectral components do not match those one would expect from the partials of a harmonic spectrum (see Schneider 1997:437ff.).

Finally, one has to take the clapper or hammer as well as the impact which sets the bell into vibrational motion into account. The clapper or hammer transmits a force via the impulse. As can be expected, there are relations between the mass of a clapper and the duration (in ms) of the contact time between clapper/hammer and bell. Also, the force *F*(t) transmitted by the impact depends on the impedance *Z* of the bell (for flexural vibrations which are of concern). In earlier experiments it was found that, roughly, mass of the clapper and contact time are proportional, that is, doubling the mass of a clapper means approximately doubling the contact time with the bell. This in turn leads to diminishing amplitudes of higher partials in the sound spectrum (cf. Grützmacher et al. 1965/66, 41-43). Impact dynamics in carillon bells recently have been investigated in detail (Fletcher, McGee & Tarnopolsky 2002). It was found that for an impedance of a bell of $Z = 3 \times 10^4$ kg s^{-1} (realistic for a bell of given dimensions) the clapper comes to rest against the bell's wall, and is pushed back then by a returning vibrational pulse. Hence, contact time is so that maximum energy transfer to the bell can take place. Since the contact time is of influence on the spectral energy distribution, short contacts account for „brighter" sounds. It has been suggested that re-voicing of carillon bells (whereby the original curvature at the site of the clapper/hammer impact is restored) is suited (a) to increase the impact duration so that the sound becomes more „mellow" (and less inharmonious), and (b) to make the impact time more dependent on impact velocity whereby „strong" playing (Dutch: *sterke slag*) makes notes both louder and brighter (the same effect is observed when playing a piano at various dynamic levels).

In fact, the carillonist by his or her way of playing can influence the force transmitted by an impulse to a bell to a considerable degree. It is thereby possible (within certain limits) to control the number of modes of vibration that are actually elicited as well as the decay of partials. It should be noted, in this respect, that (internal as well as acoustical) damping for at least some of the normal modes in bells seems to be rather small for the decay time in particular of the hum can be very long. For one famous German swinging bell (the *Gloriosa* of Erfurt, cast in 1497 by Gerardus de Wou) which, to be sure, is very large (diameter = 2570 mm, weight = 11450 kg), the sound is reported to be audible for 310 seconds (see Schad & Frik 1993b, 113). As a rule of thumb, for larger bells a decay time (60 dB from initial level) of the partials dominant in the sound (in particular the prime and the minor third) of $t_d > 10$ seconds seems reasonable. The hum, though usually being weak in amplitude, would be audible much longer.

For actual performances of music on carillons this implies that there is a temporal as well as a spectral overlap of complex inharmonic sounds likely to cause ambiguity as to pitch perception and recognition of melodic and harmonic textures. Ambiguity of pitch (and also timbre) can result, though, from a single (carillon) bell sound due to the inharmonicity of the spectrum, distribution of spectral energy, modulation effects and, to be sure, interaction of a virtual pitch (the strike note) with the pitch(es) of single spectral components.

3. Sonological Analysis

The aspects relevant to perception that have been mentioned can, at least in part, be investigated by analysis of single sounds recorded from carillon bells as well as from combinations of such sounds either produced by a carillonist, or by playing pre-recorded sounds from a sampler, software sequencer etc. We will attempt to give a description of the spectral composition as well as of dynamic features of sounds radiated from bells, and to relate the findings to perceptional issues.

Further, in addition to other methods, sonological analyses can reveal spectral and temporal characteristics of bell sounds which reflect patterns of vibration of the bell from which sound is radiated. In order to trace sound characteristics back to the acoustic behaviour of a radiator, certain assumptions as to the relationship of the velocity of the radiator and the sound pressure variation recorded at given points in a sound field have to be made (see Möser 1988, ch. 1.2). The relationship can be fairly easy explained in case the surface of the radiator is plane. For bells, because of their compound shape, sound radiation is much more complex (see Grützmacher et al. 1965/66). In this respect, the recordings made by us on location (we had access to carillons housed in belfries and churches of Brugge, Gent, Kortrijk, Tielt, Wingene, and Hamburg-Othmarschen) cannot claim to have captured all wave components radiated from the bell's curved surface. Moreover, many of the recordings obtained on location notwithstanding all efforts have picked up some background noise (e.g., street traffic). In general, the level (dB) of noise is much below that of the signals (bell sounds) to be analyzed so that spectral components representing major modes of vibration are not difficult to identify. Also, in long-term average spectra (LTAS) as have been calculated for a reasonable number of bell sounds, the noise „floor" can be clearly separated from the spectral components, and often almost disappears.

All sounds were recorded with either a single condensor mike (Neumann U 67) or a pair of two matched mikes on DAT at 48 or 44.1 kHz/16 bit linear. Bell sounds were recorded from various positions close to the bell, and in a few instances also inside large bells as well as from under the rim. To bring the signal up to the level needed for 16 bit resolution, a preamp (V 72a, flat frequency response from 20 Hz to 16 kHz) with 32 dB gain was used. The sounds were digitally transferred into a computer. The methodology employed for the analyses in the main is DFT (see Marple 1987) or other techniques based on FFT routines such as the phase vocoder implemented in software such as *sndan* (Beauchamp 1995). Details of the evolution of sounds have been investigated also by means of prediction algorithms (see Markel & Gray 1976, McAulay & Quatieri 1986, Marple 1987). Phase plane trajectories and autocorrelation functions for onsets of sound have been calculated using *Mathematica*. To evaluate auditory effects of certain sounds, a complex-valued wavelet gammatone filter bank approach (cf. Solbach et al. 1998) was employed (see below).
The sounds that are investigated in this paper belong to bells of the historical carillon of the belfry of Brugge that was founded by Joris Dumery in the years 1742 – 1748, and to the carillon of the St. Maartenskerk in Kortrijk the 49 bells of which have been cast by A.L.J. van Aerschodt (1865), S. van Aerschodt (1880), M. Michiels (1955), and P. Sergeys (1974) (see Lehr et al. 1991). The Brugge carillon comprises 47 bells, the largest of which has a diameter of 204.7 cm at the lip, and a weight of 5.381 kg. In the course of restoration of the carillon in 1968/69, bells nos. 27 - 47 have been replaced by bells from the Dutch Royal Eijsbouts foundry constructed of course so as to match the old bells. We will be concerned here only with bells of Dumery that did not undergo changes in geometry, weight, or tuning during the process of restoration. From the Kortrijk carillon, sounds recorded from bells cast by the van Aerschodt foundry have been used .

To begin with, a typical bell spectrum shall be presented. Figure 2 shows the

amplitude spectrum (0 – 2.85 kHz) of the onset of a sound recorded from the bass clock of the Brugge carillon. The bell was set to vibration with medium force (the carillonist played mf). Besides the prominent components known from the scheme of partials (e.g., hum, prime, tierce, etc.), in the spectrum many more can be found. Some of these higher components are relatively strong in amplitude, and last for several seconds before dying out. In a LTAS calculated from 131072 samples (for a sampling rate of 44.1 kHz, this is almost 3 seconds of sound), nearly 50 components are found in the spectrum. The data listed in table 2 contain (a) the frequency components (nos. 1 – 48), (b) the name of partials (as far as such can be clearly identified), (c) the frequency (Hz) of spectral peaks (calculated by means of parabolic interpolation) and (d) relative amplitude (dB). It should be noted that the amplitudes (magnitude of components) in a DFT are slightly distorted from the weighting function (the „window", see Marple 1987, Möser 1988) that is applied to minimize the effect of truncating the signal. This doesn't affect, though, the amplitudes of components as viewed in relation to each other. For the present analysis, a Hanning window and a zero-pad-factor of 2.0 were used.
Table 2 also contains (e) the frequency ratio components have with the hum (=1), and (f) the intervals between components expressed in cents for the components 1 – 23 as well as (g) cumulated cents for these components. The sign = denotes a degenerate pair of eigenmodes which in the spectrum can be identified by two peaks being only a few cents apart.

<u>Table 2: Brugge, bell no. 1: Spectral components, frequencies, etc.</u>

A (nr)	B (name)	C (Hz)	D (dB)	E (Ratio)	F (Cents)	G (cum. Cents)
1	hum	97.2543	-47.5	1	0	0
2	prime	195.7151	-43.8	2.0124	1210.5	1210.5
3	tierce	233.4328	-54.5	2.4002	307.5	1518
4	quint	294.2532	-63.7	3.0256	400.8	1918.8
5	nominal	390.8646	-45	4.019	491.5	2410.3
6a	tenth	488.3906	-75.5	5.0218	385.7	2796
6b	=	492.8655	-59.5	5.0677	15.8	2811.8
7	eleventh	514.9921	-53.7	5.2953	76	2887.8
8		525.2246	-56.8	5.4005	34	2921.8
9a		573.3524	-80.3	5.8954	151.8	3073.6
9b	=	576.2620	-85	5.9253	8.7	3082.3
10	twelfth	589.4107	-35.9	6.0605	39	3121.3
11	thirteenth	627.5936	-66	6.4531	108.6	3229.9
12		683.7274	-98.8	7.0303	148.3	3378.2
13		712.6741	-92	7.3279	71.8	3450
14		746.296	-86.3	7.6736	79.8	3529.8
15a	double oct.	818.4632	-43.8	8.4157	159.8	3689.6
15b	=	824.0592	-92	8.4732	11.8	3701.4
16		831.5786	-79.9	8.5506	15.7	3717.1
17		851.6882	-62.5	8.7573	41.4	3758.5
18		881.3533	-73.2	9.0624	59.2	3817.7
19		916.0421	-68	9.419	66.8	3884.5
20		929.4989	-71.7	9.5574	25.2	3909.7
21		980.1413	-80	10.078	91.8	4001.5
22	upper fourth	1072.189	-52.1	11.0246	155.4	4157.9
23		1090.783	-87.5	11.2158		

24	1116.460	-85	11.4798
25	1126.178	-89.3	11.5797
26	1136.946	-78.6	11.6905
27	1176.248	-92.7	12.0945
28	1196.129	-84.7	12.294
29	1268.954	-80.2	13.0478
30	1329.422	-79.8	13.6695
31	1344.131	-66	13.82
32	1413.11	-85.5	14.53
33	1424.699	-91.7	14.649
34	1503.746	-90.3	15.462
35	1611.902	-83.8	16.574
36a	1628.955	-72	16.749
36b	1630.70	-90.9	16.767
37	1639.903	-94.8	16.862
38	1673.426	-97.6	17.207
39	1740.353	-92.6	17.895
40	1797.642	-94.5	18.484
41	1850.694	-89.5	19.029
42	1901.988	-94.9	19.557
43	1922.855	-75.5	19.771
44	2162.173	-93.9	22.232
45	2220.677	-81.6	22.834
46	2496.019	-94	25.665
47	2521.589	-79.1	25.928
48	2822.575	-90	29.023

In this table, a considerable number of rather weak components have been included in order to document the richness of the spectrum. Besides some clear harmonic relations between partials, there are also inharmonic components. For example, the double octave (component no. 15, one of the strong peaks in the LTAS) is sharp against the nominal by 79.3 cents whereas the octave between prime and nominal is perfect, and the octave between hum and prime slightly sharp (+10 cents). The tierce is less than 10 cents flat (compared to the just minor third 6/5), the major third between tierce and quint is ca. 14 cents sharp (against 5/4 ~ 386 cents), yet the quint is almost perfect in relation to the prime as indeed is the tenth. These partials seem to be very well tuned. Of the higher components, some are quite inharmonious. For example, no. 22, a relatively strong upper fourth which is flat against the double octave which itself is much too sharp. However, against the correct position of the double octave (=2400 cents above the prime), the upper fourth is sharp by 50 cents. Also, there are several components closely spaced along the frequency axis thereby accounting for some modulation. The strike note of this bell is reported to be g0 (g0 or g ~ 196 Hz is G_3 according to USA standard)[1]. As in many bells, this correponds to the frequency position of the prime.

Taking a look at the dynamic features of the bell sound under investigation, the onset of some of the prominent partials as well as their evolution over 4.9 seconds is portrayed in figures 3 and 4, respectively. The hum and the prime are quite stable in amplitude whereas the tierce first decreases in amplitude, and then rises

[1] Documentation material submitted to Marc Leman in 1998 by Koninklijk Eijsbouts (the foundry that did a restoration of the Brugge carillon in 1968, including the casting of 20 new bells).

again (fig. 3). The nominal and the twelfth decay very slowly (fig. 4) as does the overall (rms) amplitude of the sound (fig. 5). Peaking at ca. 76 dB shortly after the onset, the rms amplitude is still at about 58 dB after almost 5 seconds. A plot of all spectral components against time (figure 6) shows that within the first 2 seconds of sound, only a few of the higher components disappear. After ca. 4.7 seconds from onset, the following partials are found:

Table 3: Main components found after ca. 4.7 seconds from onset

Hz	dB	partial	Hz	dB	partialt
97.32	-52.5	hum	492.92	-79	tenth
195.78	-55	prime	515.09	-76	eleventh
233.22	-47	tierce	589.39	-51	twelfth
294.51	-79	quint	818.45	-65	double octave
390.84	-73	nominal	1072.15	-82	upper fourth

From these partials, the hum, the prime, the tierce, the twelfth, and the double octave (only weakly) are still present in the spectrum 10 seconds after the onset (fig. 6).

One aspect that can only be briefly addressed here is that of the fine structure of transients found at the onset of bell sounds. Such transients might possibly reflect nonlinear behaviour. By definition, in a linear system the principle of superposition holds. According to Perrin and Charnley (1973, 413) who had surveyed the literature on acoustics of bells up to about 1972, no evidence of non-linear vibrations had been reported. Non-linear vibrations have been observed, however, in various types of plates and shells (cf. Kalnins & Dym 1976; Leissa 1993a, 303ff., Leissa 1993b, 219ff.) some of which are used as musical instruments, e.g., gongs and cymbals (see Rossing & Fletcher 1983, Fletcher 1985, Legge & Fletcher 1989). For certain gongs the nonlinearity results in pitch shifts in such a way that with increasing vibrational amplitude the vibrational frequency shifts either up (flat plates) or down (spherical shells). Dependency of frequency on amplitude is a typical yet by no means the only nonlinear effect found in vibrating systems (see, e.g., Moon 1987, Cartmell 1990). Certain nonlinearities are due to internal stress-strain relations or other factors having to do with the structure or the material of a vibrating system.

One of the causes for nonlinear vibration discussed frequently is large-amplitude deflection. Nonlinear behaviour can occur if the force $F(t)$ applied to accelerate a vibrating system exceeds certain limits (so that Hooke's law $F = -D\,y$ with D = spring coefficient will be violated). Accordingly, the restoring force will not be linearly proportional to the displacement. The so-called Duffing-oscillator (see Moon 1987, Jackson 1989) is a well-known model for an oscillator with a non-linear (cubic) restoring force.

In bell sounds, typically there are a number of spectral components the frequency ratios of which are inharmonious relative to the hum (and/or the prime). Because of the inharmonicity of spectral components, the time function of many bell sounds shortly after the onset (before damping of higher components sets in) appears to be quite irregular so that any periodicity inherent in such signals may be difficult to detect by autocorrelation or other methods. Since the period T (the time after which a pattern of vibration repeats itself) of inharmonic sound signals can be very much extended compared to harmonic sounds (where the frequency ratios are f_n =

n x f_1, n = 1, 2, 3, ..., k), inharmonic sounds are regarded as being quasi-periodic (the „exact" period can be infinite; see Schneider 2000).
For the transient portion of many bell sounds, a complex pattern of vibration can be observed which appears to be more or less irregular. This implies it is non-periodic. Two examples shall illustrate these phenomena. One is the sound of the Brugge bell no. 1 as has been analyzed above, the other is from bell no. 2 of the famous Hemony carillon of Gent. For both sounds, the time function for the first 60 ms is given along with a spectral peak picking analysis (see Markel & Gray 1976, Marple 1987) based, in this case, on a Wigner transform (see Yen 1987) which permits a better time/frequency resolution than does the conventional FFT technique. Figure 7 shows the sound of Brugge bell no. 1 subjected to a special algorithm (Momose 1991) which computes the geometric mean between the FFT power spectrum and the Wigner transform thereby masking most of the unwanted cross-terms resulting from the Wigner transform (the cross spectrum is the sum and difference of the Fourier spectrum of the signal). Numbers 1 – 8 on the right side of the graph indicate major components: 1 = hum, 2 = prime, 3 = tierce, 4 = nominal, 5 = tenth, 6 = twelfth, 7 = double octave, 8 = upper fourth. It becomes evident that within the first 20 – 30 ms the sound is unstable in that several of the components undergo frequency shifts (which later turn into more regular modulation as far as the twelfth and double octave are concerned). In particular the hum exhibits these frequency glides over 60 ms. The unstable pattern of vibration is reflected in some sudden change of direction in the phase plane trajectories (velocity x' ./. displacement x; see figure 8) calculated from the first 2079 samples corresponding to ca. 47 ms of the sound.
The effect of frequency shifts and modulation similarly is found in the sound recorded from bell no. 2 at Gent[2]. Again, 60 ms of sound subjected to a Wigner transform plus peak picking trajectories are shown in figure 9. Numbers 1 – 6 correspond to the hum, prime, tierce, nominal, twelfth, and double octave, respectively. To illustrate the development in the phase plane, two pictures are given: the first (figure 10a) shows the state of the system for the first 180 samples, or 3.75 ms, the second (figure 10b) is calculated from 1000 samples (20.83 ms sound at 48 kHz sampling). According to fig. 10a, the vibration sets in quite regularly[3] but soon goes into more complex patterns as becomes evident from fig. 10b. The irregular patterns found in these trajectories are indicative of nonlinear behaviour (cf. Moon 1988, 4).

In comparison to the bell sounds from Brugge and Gent that have been under review so far, a few sounds from bells of the carillon of the St. Maartenskerk at Kortrijk shall be analyzed. Whereas the carillons of Gent (P. Hemony) and Brugge (J. Du Mery) are deemed being representative of the 17th and 18th centuries, respectively, the carillon of the St. Maartenstoren which contains many bells cast by Andreas Lodewijk van Aarschodt and Severinus van Aarschodt is said to reflect „romantic" ideals of sound. Rather critically ist has been remarked that these bells are from a time *dat men wel mooi klinkende, maar geen zuiver klinkende klokken kon maken* (Lehr et al. 1991, 209 and 211). This means that the bells in question may have a nice sound yet are not well-defined in regard of pitch(es) and scale.

[2] As to the history of the Gent carillon see Lehr et al. 1991, 137ff.
[3] Periodic vibrations result in closed trajectorires (or „orbits"); e.g., the trajectory for a sine wave is an ellipse, the trajectory of a triangle wave is a rectangle (see Meyer & Guicking 1975, 374ff.).

To check this judgement, a detailed study including psychoacoustic experiments would have to be made. However, a sonological analysis which investigates the spectral composition of bell sounds can contribute to such an evaluation. Given that a certain ‚inner harmony' based on small integer frequency ratios of partials is taken as a standard (see table 1), the analysis can reveal how well partials of a certain bell match such a standard.
To give an example, frequencies and frequency ratios from sounds recorded from bells no. 1, 2 (A.L. van Aarschodt) and 3 (S. van Aarschodt) are listed in table 4. Frequency data have been obtained from FFT LTAS measurements plus exact estimate of spectral peaks (frequency, rms error of peak estimate). For bell 1, the list is almost complete (including rather small spectral components) whereas for bells nos. 2 and 3 only components up to 2 kHz have been included. For bell no. 1 also relative amplitudes are given for each of the 46 components. It should be noted that all three bells have been excited with little effort (*zachte slag*).

Table 4: Kortrijk, St. Maartenskerk, bells nos. 1 – 3, frequencies, frequency ratios

	bell 1			bell 2		bell 3	
No.	Hz	dB	ratio	Hz	ratio	Hz	ratio
1 hum	123.772	-39.5	1	132.95	1	143.716	1
2 prime	236.754	-41.1	1.913	253.155	1.904	284.401	1.979
3 tierce	286.449	-45.7	2.314	12.239	2.348	349.685	2.433
4 quint	368.275	-62.5	2.975	392.226	2.95	408.346	2.841
5	430.96	-65.4	3.475	512.635	3.856	576.123	4.009
6	466.215	-28	3.766	636.872	4.79	674.085	4.69
7a	540.522	-61	4.367	645.504	4.855	693.096	4.823
7b	548.285	-64	4.429	649.673	4.886	698.103	4.857
8	585.129	-40	4.727	758.782	5.707	704.749	4.904
9	611.136	-49	4.937	835.85	6.287	855.589	5.953
10	684.248	-39	5.528	873.248	6.568	890.817	6.198
11	788.418	-50	6.37	934.81	7.031	984.596	6.851
12	855.352	-78	6.91	1036.548	7.796	1016.95	7.076
13	884.553	-77	7.145	1052.036	7.913	1094.897	7.618
14	931.287	-30.5	7.524	1053.758	7.926	1174.688	8.174
15a	956.026	-53	7.724	1076.145	8.094	1203.938	8.377
15b	963.735	-56	7.786	1132.594	8.519	1223.45	8.513
16	988.369	-52	7.985	1185.836	8.919	1258.927	8.76
17	1065.848	-66	8.611	1340.588	10.083	1528.173	10.633
18	1117.274	-52	9.027	1416.634	10.655	1906.905	13.268
19	1153.407	-83	9.319	1419.911	10.68		
20	1178.337	-87	9.52	1484.28	11.164		
21	1200.726	-45.5	9.701	1504.868	11.319		
22	1288.287	-64	10.408	1621.976	12.199		
23	1336.107	-64	10.795	1662.876	12.507		
24a	1387.435	-73.5	11.209	1728.426	13.0		
24b	1394.245	-68.5	11.265	1814.209	13.646		
25	1418.638	-78	11.462	1835.259	13.804		
26	1484.777	-47.5	11.996	1874.661	14.10		
27	1523.49	-71	12.309	1998.804	15.034		
28	1573.252	-88	12.711				
29	1622.482	-70	13.108				
30	1644.105	-74	13.283				

31	1725.692	-73	13.942
32	1755.618	-73	14.184
33	1778.667	-57.5	14.37
34	1863.479	-90	
35	1954.838	-83	
36	1974.921	-85	
37	1985.092	-83	
38	2078.796	-75.5	16.795
39	2147.102	-78	
40	2288.389	-86.5	
41	2308.253	-90	
42	2327.367	-82	
43	2380.629	-71	19.234
44	2684.279	-83.5	
45	2987.204	-81.5	
46	3284.635		

In this table, „a/b“ refers to nearly degenerate pairs found in bell no. 1. Accidentally, components 7a/b were found to be split modes also in bell no. 2, and possibly also in bell no. 3 (where the respective components are only 12.5 cents apart). In bell no. 2, an almost degenerate pair consists in components nos. 18 and 19 which are only 4 cents apart.
The data for bells no. 1-3 show that some of the partials indeed approximate frequency ratios of an „ideal“ minor third bell whereas others deviate considerably. In general, in historic carillon bells partial frequencies vary from one bell to the next so that actual data match the scheme given in table 1 more or less. For example, in some bells one will encounter minor thirds which are much too flat (e.g., Mespelare near Dendermonde, Flanders; cf. Schneider 1997, 440) whereas in others the minor third is too sharp as is the case with bell no. 3 from Kortrijk. The minor third in this bell is 357.8 cents above the prime, and hence a „neutral“ rather than a minor third. Though a neutral third might be suited to render musical pieces in both the minor and major mode, it seems that both carillonists and laymen do not appreciate such a tuning of this partial (cf. Houtsma & Tholen 1987).
A real problematic sound in the Kortrijk carillon comes from bell no. 4 where many twin peaks are found in the spectrum (see figure 11) indicating warble. Typically, this is caused by deviations of the bell corpus from axial symmetry. The main components which are contained in the sound of bell no. 4 (excitation done with medium force: mf) are listed in table 5.

Table 5: Kortrijk, bell no. 5, main spectral components

No.		Hz	dB	ratio
1a	hum	151.175	-39.2	1
1b	=	154.414	-53.8	1.021
2	prime	307.805	-35.5	2.036
3a	tierce	357.009	-41.1	2.361
3b	=	363.821	-29.5	2.406
4a	quint	489.832	-60.13	3.24
4b	=	491.969	-60.13	3.25
5a	nominal	600.346	-52.94	3.971

5b	=	611.474	-46.23	4.045
6a	tenth(?)	816.796	-83.26	5.403
6b	=	819.819	-82.33	5.423
7		835.116	-84.45	5.524
8a	twelfth	907.634	-79.67	6.003
8b	=	921.296	-33.68	6.094
9a		1065.541	-78.91	7.048
9b	=	1069.891	-65.93	7.077
10a		1260.927	-71.71	8.341
10b	=	1265.643	-60.97	8.372
11		1279.356	-42.27	8.463

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There are some more twin peaks in the spectrum at higher frequencies. The many pairs of narrowly spaced components result in amplitude modulation of ca. 12 Hz (figure 12) which is clearly audible. Because of the modulation, the bell has a shimmering sound.

4. Psychoacoustic considerations

The amplitude modulation due to warble in the bell typically goes along with yet another effect, namely changes in the spectral centroid which is a weighted mean (or rather, median) of the spectral energy distribution (see *sndan* docu; Beauchamp 1995) suited to describe the ‚brightness' of a sound. In this respect, the physical phenomenon of nearly degenerate pairs of eigenfrequencies often leads to two psychoacoustic effects, (1) more or less periodic changes in the sound pressure level due to amplitude modulation as well as (2) more or less rapid changes in spectral centroid. For bell no. 4 of the Kortrijk carillon, the first effect is documented in figure 12, the second in figure 13. It has been pointed out earlier that also frequency modulation of partials occurs in particular at the onset of sounds (cf. figs. 7, 9). Even in historic bells considered to represent very high standards of craftmanship one can find some shifts in partial frequencies as well as more regular modulation. For example, bell no. 8 from the Brugge carillon (J. Du Mery 1744, 1145 kg) when set to vibration with strong excitation (*sterke slag*) produces a sound in which some of the spectral components show frequency modulation which however is slight (for a pitch tracking McAulay/Quatieri [1986] analysis of the hum, see figure 14). Such small changes in frequency will not affect pitch perception yet will account for the dynamic („living") quality of bell sounds.
Another aspect worth mentioning is spectral energy distribution with respect to the so-called dominance region for virtual (residue) pitch (cf. de Boer 1976) as well as in regard of the sensítivity of our sense of hearing (see Terhardt 1998). Regarding the „dominance region" for virtual pitch, it can be said that of all the partials of sufficient strength those found in the range of, roughly 400 – 2000 Hz will be the most important ones to contribute to a virtual pitch (e.g., the strike note of a bell; see Fleischer 1996).
Especially in sounds resulting from strong excitation of the bell, one can find many components corresponding to eigenfrequencies and eigenmodes of the bell in this frequency range. For example, in the sound of bell no. 8 from Brugge about 20 components several of which are quite strong in amplitude (see figure 15), are spaced along the frequency axis from 392.5 Hz (tierce) to ca. 1.9 kHz. The

relevant spectral components of this sound which contains spectral energy up to 5.2 kHz are listed in table 6 (FFT: 131072 pts, Hanning).

Table 6: Brugge, bell no. 8, spectral composition and frequency ratios

No.	partial	Hz	dB	ratio	cents	cents cum
1	hum	163.9779	-49.5	1	0	0
2a	prime	326.3417	-44.2	1.9901	1191.2	1191.2
2b	=	327.7384	-51.5	1.9987	7.6	1198.8
3	tierce	392.4982	-36.2	2.3936	312.2	1511
4	quint	493.2762	-56.87	3.0082	395.7	1906.7
5	nominal	659.8955	-52.93	4.0243	503.8	2410.5
6		811.557	-71.44	4.9486	358.1	2768.6
7a	tenth	823.8167	-58.73	5.0239	25.9	2794.5
7b	=	825.9812	-58.85	5.0371	4.5	2799
8		844.0778	-75.73	5.1475	37.5	2836.5
9a	eleventh	849.3215	-50	5.1795	10.7	2847.2
9b		852.0538	-61.1	5.196	5.5	2852.7
10		930.775	-82.2	5.676	153	3005.7
11a	twelfth	997.4539	-44	6.0828	119.8	3125.5
11b	=	1000.538	-71.7	6.101	5.2	3130.7
12	thirteenth	1060.711	-51.8	6.4682	101.1	3231.8
13	double oct.	1323.228	-61.1	8.0695	382.7	3614.5
14		1383.988	-48.2	8.44	77.2	3691.7
15a		1411.234	-67	8.6062	33.8	3725.5
15b		1417.159	-73.8	8.6424	7.2	3732.7
16a		1439.799	-62.6	8.7804	27.4	3760.1
16b		1441.611	-67.6	8.7915	2.1	3762.2
17		1447.186	-59.6	8.8255	6.7	3768.9
18		1522.842	-63.2	9.2869	9.1	3778
19a		1557.97	-79.7	9.5011	39.5	3817.5
19b		1560.362	-85	9.5157	2.7	3820.2
19c		1563.399	-82	9.5342	3.3	3823.5
20		1608.239	-72.5	9.8076	49	3872.5
21	upper fourth	1809.932	-47.7	11.0376	204.6	4077.1
22		1882.343	-69.4	11.4792	68	4145.1
23		1952.623	-85.4	11.9078	63.5	4208.6

Above 2 kHz, some more relatively strong components are found, two of which are

27a		2263.077	-71.2	13.8011		
27b	=	2265.988	-57.5	13.8188		4545.8
24	triple oct.	2740. 348	-61	16.7117		4874.8

Estimation of the strike note based on subharmonic matching of those strong partials which almost fit into a harmonic series (nominal, tenth, twelfth, double octave, upper fourth) yields two „candidates", one in the frequency range of the prime at about 330.5 Hz, the other slightly above the hum (ca. 165.3 Hz). This would conform to findings from experiments which employed sounds from both church and carillon bells. For sounds from historic church bells it was found that almost all bell sounds (97% from 137 bells) gave rise to the sensation of at least

one strike note; however, in about 69% of these sounds, only one strike note was sensed whereas ca. 30% were ambiguous as to the strike note. From experiments in which subjects were asked to match the frequency of a sine wave generator to the strike note (whereby the strike note is considered to constitute the „main pitch" of a bell) of a bell sound heard as a stimulus, three categories of strike note locations can be distinguished: in 12% of the bell sounds, the strike note was identical with a spectral pitch (the frequency of a partial found in the spectrum), for 77% the strike note was close to yet not identical with the frequency of a partial, and for 3% the frequency corresponding to a strike note was not near the frequency of any spectral component (cf. Terhardt & Seewann 1984). In particular „naive" listeners (having little or no formal musical training) are uncertain about the octave, that is, they tend to confuse the frequency position around the hum with that of the prime. To be sure, „octave mistakes" are by no means peculiar to bell sounds or restricted to „naive" listeners. To the contrary, such mistakes are often made by subjects claiming to possess „absolute pitch" (see Heyde 1986). They tend to correctly identify the tonal category by its note name (e.g, c, d, f#, b^b) yet often fail to identify the correct octave. One reason why „octave mistakes" occur even for harmonic sounds (as produced by wind and string instruments) is that in certain sounds radiated from musical instruments the fundamental is much weaker than the second harmonic. In bells, due to the partly harmonic, partly inharmonic spectral structure, ambiguity regarding the „main pitch" is frequently encountered. Subjects not only have doubts about the „correct" octave of the strike note yet also may perceive two different strike notes per stimulus, one being located at or close to the prime, the other about a fourth above the prime (for valuable experimental data concerning strike notes, see Bruhn 1980). Since also some of the partials can be heard separately as spectral pitches, in particular the tierce which in many sounds is strongest in amplitude, ambiguity as to the „main pitch" of a complex bell sound is a common experience even for musically trained subjects.
In experiments in which carillon bell sounds were employed as stimuli, results similar to those obtained from church bells have been reported. Subjects asked to match the pure tone from a sine wave generator to carillon bell sounds[4], chose various partials as a „main pitch", namely the prime, hum tierce, quint, and double octave, respectively. For three bells taken to represent three octaves of a carillon, the „main pitch" according to the partials preferred by subjects in matching sine tones is as follows (Fleischer 1996, 23ff., 31):

Table 7: Partials chosen as „main" pitch of a bell sound (% of frequency matches)

Partial	bell $A^b{}_5$	bell $F^\#{}_6$	bell D_7
Hum	8%	19%	29%
Prime	30%	28%	19%
Tierce	4%	2%	1%
Quint	4%	1%	0%
Nominal	8%	0%	0%

In regard of the „main pitch", the prime obviously is the most important partial in

[4] The experimental design in these and many other experiments is that subjects hear the stimulus (in this case, a bell sound reproduced by means of earphones), and after a short interval of, usually, a few seconds start to manipulate the sine wave generator in order to produce a frequency being „equal in pitch".

the lowest of the three octaves. Inasmuch the importance („pitch strength"or salience) of the prime descreases in higher octaves, the importance of the hum increases. This of course has to do with the „dominance region"(to be sure, D_7 with respect to the prime means a frequency of 2349 Hz) as well as with perception of *musical* pitches in general. Musicians are trained to make pitch estimates, most of all, in the range from, about, A_2 to C_6 (A to c''', that is, for fundamental frequencies from 110 to 1046.5 Hz). Partials which fall into this range can be expected to be preferred in pitch matching tasks that were employed in the experiments under review. Worth mentioning is that a number of subjects for some of the bell sound chose a frequency as „main pitch" that was slightly above the frequency of either the prime or the hum (cf. Fleischer 1996, 23ff.). The possible explanation to this finding is that these subjects tried to match the sine wave to a strike note that, as a virtual pitch derived from higher partials, comes close to either the prime or hum yet doesn't exactly fall onto the same frequency.
For the sound from bell no. 8 of the Brugge carillon (see above), a similar behaviour can be predicted due to the frequencies and amplitudes of partials most relevant to formation of a strike note located close to the prime or hum. However, the spectral structure of this sound (see table 6) documents another factor relevant to pitch and timbre perception, that of spectral density. Evidently, there are several components (besides the nearly degenerate pairs, e.g., 2a/b, 7a/b, etc.) closely spaced along the frequency axis (e.g, nos. 6 and 7a/b, 7a/b to 8, 8 to 9a/b, and so on). According to psychoacoustic concepts, it can be assumed that such components in general will „mask" each other, and that of components being so close in frequency as to falling into the same „critical band" (see Hartmann 1998, ch. 10), only those might become audible which are significantly stronger in amplitude than are their neighbours. For example, component no. 14 at 1384 Hz exceeds its neighbour 15a/b by about 19 dB. Even if spectral components do not become audible individually, they often contribute strongly to the harmonicity or inharmonicity of a sound as well as to its timbre. In western church and carillon bell sounds, there are partials which are clearly audible as such (as is evident in particular for the minor third which often is the strongest partial in a given spectrum as in, for example, Brugge no. 8). Besides, there are components which may be difficult to „hear out" yet which influence the overall quality of a sound in regard of ambiguity of pitch due to spectral density and interaction of components (roughness or beats due to amplitude modulation) as well as with respect to sharpness (see Aures 1981) and a sometimes metallic timbre. In fact, different from harmonic sounds where, typically, the fundamental accounts for the „pitch", and the number and relative strength of harmonics for the „sound colour" (as was explained in classic publications by H. von Helmholtz and C. Stumpf), in many if not most inharmonic sounds there is no clear distinction between pitch and timbre (see Schneider 1997, 2000). In principle, this holds true also for bell sounds which because of the partly harmonic, partly inharmonic spectral structure by many listeners are judged to be ambiguous with respect to pitch(es) (see Bruhn 1980, Terhardt & Seewann 1984, Fleischer 1996). Ambiguity of pitch in part is reflected in the lack of clear periodicities in the time series of bell sounds as measured by autocorrelation. Moreover, because the minor third in general stands out against the strike note (in most cases identical with, or close to, the prime) and its upper (nominal) and lower (hum) octaves, it is perceived as a second tonal element (indeed, as a minor third which forms a musical interval with, typically, the prime

and/or strike note). In effect, a complex bell sound may be sensed as a sonority rather than forming a highly coherent musical note. This is of consequence as regards intervals and scales (see below). It must be said, though, that sounds obtained from individual bells rarely give rise to the sensation of roughness to a significant degree (see Aures 1981). Sounds recorded from the carillons of Brugge (J. Du Mery) and Hamburg-Othmarschen (J. Schilling, Apolda 1933) which were subjected to a computer algorithm that measures roughness according to a psychoacoustic model (Daniel & Weber 1997), did not exceed the level of 0.1 asper[5] except for the very attack of sounds (where the clapper/hammer hits the bell) and strong playing (*sterke slag*). In case the sound contains considerable amplitude modulation due to spectral density and/or inharmonicity of components, this process of course is reflected in the respective roughness pattern (see figure 16, Othmarschen, bell no. 3, weak excitation, heavy modulation, compared to bell no. 5, strong excitation, little modulation).
Roughness seldom appears as a significant perceptual factor in a *single* bell sound (with the exception of poorly cast or damaged bells). One has to take into account, though, the situation where real music including rich melodic textures and chords is performed, often in a fast and virtuoso manner. Because of the melodic embellishments and chordal structures which typically are executed as arpeggios, quite many complex sounds may be produced per time span (say, 10 seconds of a given performance). Because of the low degree of damping observed for each bell sound (see also fig. 16), on the one hand, and the partly inharmonic structure of bell sounds, on the other, the overall spectrum resulting from the music can be very dense, and difficult to analyze perceptually. The spectral energy distribution has to be viewed with respect to integration intervals of about one second each that are needed for feature extraction from the acoustic signal (achieved with the aid of the so-called „echoic memory", see Snyder 2000). The features are then bound together to constitute musically relevant „building blocks". The problem is that the information available from actual spectral distributions can be ambiguous even for musically simple structures like, for example, a C-major chord. Analyzing a famous work of jazz music performed on the Brugge carillon (*Misty*, written by Erroll Garner, performed by Aimé Lombaert[6]), we found that in a C-major chord no less than 42 strong to medium strong spectral components are contained in the frequency range from 195.76 Hz (G_3 or g) to 2778.5 Hz (see Schneider & Müllensiefen 1999, Tab. 3). Several of the components corresponding to partial notes such as B_5 (h'') or D flat $_6$ (des''') are highly dissonant in the given chord, and „blur" the tonal meaning of such chords within a harmonic structure. The same holds true with respect to polyphonic settings. Even for musically trained listeners (yet unexperienced with carillon music), renditions of western art music or of jazz on a carillon appear to be demanding in terms of apperception of tonal structures since there is an „overload"of information induced from the dense and partly inharmonic spectra (see Schneider 2001). The perceptual task to deal with these unfamiliar and often ambiguous sounds, as well as with musical textures based thereon, also has an effect on appreciation of carillon music. In experiments where a harmonized melody performed in two versions (minor and major mode) was

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[5] The unit 1 asper is defined by the roughness evoked by 100% amplitude-modulated 1 kHz tone at 60 dB at a modulation frequency of 70 Hz (Daniel & Weber 1997, 114).
[6] This version is found on the CD *Emanation. Waves of music, water and wind.* Aimé Lombaert and the Noordzee Brassband Brugge. Digi Classics Intern. Productions [MBZ 01] 1995.

played on a (partly synthesized) carillon which alternately had a minor, a major as well as a neutral third, it was found that carillon students prefer minor-third bells no matter what the mode of the musical piece is. Other music students opted for a correspondance of mode and bell tuning so that a piece in a minor key should be played with minor-third bells, a piece in a major key with major-third bells. Nonmusicians preferred major-third bells in both versions. None of the three groups seemed to esteem „neutral" thirds (see Houtsma & Tholen 1987).
From these findings one could infer that, in order to meet the taste of a wider audience, major-third bell carillons should be used in particular to perform music written in a major key. In fact, the major-third bell significantly reduces the inharmonicity found in all idiophones, and thereby also the burden of spectral information to be processed by our sense of hearing. On the other hand, one may argue that some ambiguity in regard of pitch, pitch relations as well as timbre experienced in music performed on a minor-third carillon constitutes a perceptual quality in itself which can be appreciated[7]. One aspect that would deserve more detailed investigations seems to be the *degree* of ambiguity which can be considered as desirable or allowable with respect to apperception and appreciation of tonal music played on carillons. Since this ambiguity is largely dependent on the degree of inharmonicity of bell spectra, it follows that some positive correlation can be expected between increasing inharmonicity (which implies increasing complexity of perceptual tasks in regard of pitch and pitch relations) and decreasing values on variables pertaining to apperception and appreciation of music (see Schneider & Müllensiefen 1999, Schneider 2001).

Finally, some aspects of scales should be addressed. In case one considers the prime to be the decisive partial in regard of pitch and scale formation, the first octave (bells no. 1 – 11) of the Brugge carillon comprises the following intervals:

bell no.	1	2	3	4	5	6	7	8	9	10	11
cents	190.4	209.7	107.6	72.1	126.3	68.6	110.8	114.9	66.3	129.5	
cumul.	0	190.4	400.1	507.7	579.8	706.1	774.7	885.5	1000.4	1066.7	1196.2
note	g	a	b	c'	c#'	d'	d#'	e'	f'	f#'	g'

The carillon is reported to be tuned in mean-tone temperament[8]. Indeed, there is a sequence of small (chromatic) and large (diatonic) semitones being characteristic for mean-tone tuning; also, the frequency distance (converted into cents) between the notes g and a comes close to the „ideal" mean-tone step (193.15 cents). The tuning as described in the scheme above is one-dimensional in that it is based on prime frequencies only. These are believed to be identical with, or close to, the strike note of each bell. A description of a tuning scheme based on strike note frequencies, though making use of perceptual data, again is one-dimensional in that each complex bell sound reduced to but one element, the strike note. This reduction process seems necessary in order to establish a *musical* scale which again is a one-dimensional construct (see Schneider 1997, 2000, 2001). On the other hand, in regard of perception, each complex inharmonic bell sound is a multi-dimensional stimulus. The ambiguity found in bell sounds as well as in scales

[7] By analogy, one may point to „bi-tonal" harmonic structures found, for example, in Stravinskij's *Petrushka*. Such harmonic textures constitute a
[8] Tuning data by Koninklijke Eijsbouts which documents the state of the carillon after ist restoration in 1968.

made therefrom perhaps rests in the fact that the reduction from multi-dimensional sounds to one-dimensional scales or melodic lines for subjects not being experts in carillon music is difficult to perform. This holds true in particular for an actual listening situation (e.g., a concert) since the processing of complex sounds has to done in real time. Naturally, the task is more demanding for polyphonic pieces where several such reductions need to be carried out in order to „detect" melodic contours for each voice.

To illustrate the basic problem, at this point one example must suffice. The sounds recorded from bells making up a scale in a carillon in general are all different with respect to spectral energy distribution even if the carillonist attempts to play with approximately equal force (say, mf). Typically, sounds in a bell scale are uneven regarding the pattern of spectral components as well as their relative amplitudes. This can result in the experience of sequences of sounds representing a musical scale yet which do not rise smoothly in terms of both ‚pitch' and ‚brightness' (as in general is the case in harmonic sounds since all the partials are shifted up the frequency axis in parallel). If we take segments of about one second each of the sounds from bells no. 1, 2 and 3 of the Brugge carillon, part of a musical scale is established, namely the notes g $\rightarrow$ a $\rightarrow$ b (see above). The relevant partials and energy distribution as derived from processing the sounds through a special gammatone filter bank (which simulates the first stage of our auditory system, see Solbach et al. 1998) are shown in figure 17 where the relative strength of partials is indicated by grey/black scales. Obviously, in the sound of bell no.1 much of the spectral energy rests with higher partials such as the twelfth and the double octave (cf. table 2 above) whereas the tierce is rather weak in this sound. In sound no. 2, the tierce is the prominent partial. The nominal in this sound is stronger than in sound no. 1. In sound no. 3, the tierce, nominal, and twelfth are strongest. Computation of the spectral centroid (which corresponds closely to the psychoacoustic parameter of brightness) for the three sound segments (figure 18) indicates that scale step 2 (note a) does not rise in ‚brightness' relative to scale step 1 (note g) yet remains on the same level (on the average: 750 – 800 Hz) or even drops down a bit. Such phenomena contribute to ambiguity which can be experienced in single bells sounds as well as in sequences of such sounds arranged so as to form a ‚scale'.

5. Summary

This article, for limitations of space available, presents detailed analyses of but a few sounds recorded from carillon bells. Temporal and spectral features of the sounds were investigated with the aim that a sonological analysis may contribute also to our understanding of the acoustics of bells. For example, by means of LTAS one can identify nearly degenerate pairs of eigenfrequencies in a precise way. On the other hand, sonological analyses can reveal characteristics of bell sounds which are of relevance in regard of perception of such sounds as well as with respect to apperception and appreciation of music performed on carillons. Ambiguity of pitch and timbre as is experienced by many subjects when listening to single carillon bell sound as well as to sequences of such sounds can be explained, in the main, from the spectral structure of bell sounds which is partly harmonic, and partly inharmonic. Since actual spectral composition and energy distribution varies considerably in carillon bell sounds, the extraction of salient perceptual features seems to be more difficult than from conventional (harmonic) sounds.

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